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# Large Quark Rotations, Neutrino mixing and Proton Decay

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## Abstract

Right-handed (RH) rotations do not play a role in the Standard Model, and only the differences of the LH mixing angles are involved in  $\mathbf{V}_{\text{CKM}}$ . This leads to the huge freedom in the fermionic mass matrices. However, that is no more true in extensions of the Standard Model. For example in GUTs large RH rotations of the quarks can be related to the observed large neutrino mixing or in particular, all mixing angles are relevant for the proton decay. We present a simple realistic non-SUSY  $SO(10)$  GUT with large RH and LH mixing and study the corresponding nucleon decay rates.

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# 1 Introduction

What is the origin of the fermionic masses? This is one of the open questions in the Standard Model (SM). The mass matrix entries are arbitrary parameters of the model and only the neutrinos are restricted to be massless.

One can add conjectures for the structure of the fermionic mass matrices to the SM. Many different conjectures are known to give the right masses of the charged fermions and  $\mathbf{V}_{\text{CKM}}$  (within the experimental errors) and this is clearly an indication that the mass problem is far from being solved.

The main reason for this large freedom is that right-handed (RH) rotations<sup>1</sup> are non-observable in the SM. Also the observed left-handed (LH) mixing matrix  $\mathbf{V}_{\text{CKM}}$  involves only the *differences* between the mixing angles of the *u*-like and *d*-like quarks and the individual mixing can be large. Actually, there is already a strong indication from the neutrino sector that large rotations are required [2, 3].

The considerable freedom in the mass matrices of the SM does not exist in its extensions. We know that the SM must be extended for many reasons if not alone to explain the fermionic masses and mixing angles. In particular, in its most popular extension, the Grand Unified Theories (GUTs) [4], all mixing angles are relevant, as will be explained later. In such a framework one cannot change the “weak basis” without changing the physics.

The aim of this paper is to present a simple realistic  $SO(10)$  GUT [5] model with large LH as well as RH mixing angles and to use them to study the consequences for nucleon decay. Mixing effects were generally neglected in the conventional proton decay models [6] by assuming that the mixing angles are small.

Large rotations in the quark sector could be a natural reason for the large mixing observed in the leptonic sector in terms of neutrino oscillations. In particular, there is a kind of duality between the RH mixing of the quarks and the leptonic LH rotations [7].

A predictive model for the fermionic masses must involve a family symmetry which dictates the texture of the mass matrices and protects it from getting large radiative corrections. We used a global  $U(1)_F$  [8] in the framework of an  $SO(10)$  GUT that will add relations between the matrix elements. Our aim was to look for the simplest possible realization of a realistic model with large mixing angles. We therefore used the famous  $SO(10)$  paper of Harvey, Reiss and Ramond [9] and generalized it to asymmetric matrices. Such matrices give usually large LH and/or RH mixing angles.

We did not use non-renormalizable contributions to the mass matrices à la Froggatt and Nielsen [10] because this method assumes ad hoc physics beyond the GUT and many new particles. In addition, the resulting matrix elements are given there in orders of magnitude only. This can explain the hierarchy of the masses but not the light see-saw neutrino properties. Those are obtained from a product of three matrices and hence predicted up to a factor of  $[\mathcal{O}(1)]^3$  which may be quite large [11].

Most recent models use SUSY GUTs [12]<sup>2</sup>. However, the available parameter space of low-energy SUSY shrunk recently so much that MSSM is on the verge of loosing its “naturalness” [14]. At the same time solutions without low energy SUSY have emerged quite naturally [15] in superstring and M theory, and also the hierarchy problem can be

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<sup>1</sup>A general non-hermitian matrix is diagonalized by a bi-unitary transformation. This means that two unitary matrices, one from the left and one from the right, are needed. Those matrices are equal only for hermitian (or symmetric) matrices [1].

<sup>2</sup>We shall use it also in a forthcoming paper [13].

solved adding extra dimensions [16]. We think therefore that it is worthwhile to look for a non-SUSY GUT which is consistent with all observed experimental facts. The hope is that the fine tuning (hierarchy) problem will be solved in the more fundamental theory. Note also that conventional GUTs are relatively simple and give much more reliable predictions for nucleon decay than SUSY theories.

Unification of the gauge coupling constants is obtained using an intermediate breaking scale [17]  $M_I \simeq 10^{11}$  GeV. This is very useful because  $M_I$  is also the right mass scale for the RH neutrinos needed for the see-saw mechanism as well as for leptogenesis as the origin of the baryon asymmetry [18] and the (invisible) axion window [19].

The observed neutrino oscillations teach us about the neutrino masses and mixing angles. This is the first evidence for physics beyond the SM. Our claim is that observation of other phenomena like RH currents, leptiquarks, baryon asymmetry induced by leptogenesis and especially nucleon decay can reveal the unknown mixing angles and reduce considerably the freedom in the fermionic mass matrices.

The plan of the paper is as follows. In section 2 we discuss the symmetry breaking that is dictated by the requirement of gauge unification and the Higgs representations needed to give the correct fermion masses. The mass matrices and our global  $U(1)_F$  details are given in section 3. In section 4 the numerical solutions for the mass matrices are obtained by the use of the renormalization group equations (RGEs), and a fit to the observed properties of the charged fermions is elaborated. Three solutions are found which are consistent with the observed neutrino anomalies [2, 3, 20, 21] (except for LSND [22]). All mixing angles for those solutions are obtained explicitly and this allows for the calculation of the nucleon decay rates in section 5. Section 6 is devoted to the conclusions.

## 2 Symmetry breaking of the $SO(10)$ GUT

We use an  $SO(10)$  GUT [5] which is broken down to the SM via an intermediate scale  $M_I$  as follows

$$SO(10) \xrightarrow{M_U} G_I \xrightarrow{M_I} G_{\text{SM}} \xrightarrow{M_Z} SU(3)_C \otimes U(1)_{\text{em}} , \quad (1)$$

where the intermediate symmetry group is the Pati-Salam one [23],  $G_I = G_{\text{PS}} \equiv SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ .

The breaking at  $M_U$  is done using the Higgs representations  $\Phi_{210}$  (or  $\Phi_{54}$  in models with  $D$  parity) while for the breaking at  $M_I$  we use the SM singlet of a  $\Phi_{126}$ , i.e. the  $G_{\text{PS}}$  representation  $(\mathbf{10}, \mathbf{1}, \mathbf{3})_{126}$ . For the masses of the Higgs scalars the “extended survival hypothesis” [24] is assumed.

In view of the representation content of the mass terms

$$\mathbf{16} \otimes \mathbf{16} = (\mathbf{10} \oplus \mathbf{126})_{\text{symm}} \oplus \mathbf{120}_{\text{antisymm}} \quad (2)$$

we give the light fermions masses via the VEVs of  $(\mathbf{1}, \mathbf{2}, \mathbf{2})_{10/120}$  and  $(\mathbf{15}, \mathbf{2}, \mathbf{2})_{120/126}$ . The RH neutrino masses of order  $M_R \sim M_I$  will be induced via the VEV of the above mentioned  $(\mathbf{10}, \mathbf{1}, \mathbf{3})_{126}$ .

The Higgs doublet of the SM is therefore a linear combination of the  $SU(2)_L$  doublets in the representations  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  and  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$ . The exact number of representations needed for the fermion mass matrices will be given later when we will discuss those matrices. This number is needed however to fix the RGEs used to calculate  $M_I$ ,  $M_U$  and  $\alpha_U(M_U)$  as well

as the values of the mass matrices at  $M_I$  <sup>3</sup>. As will be explained in the next chapter, we used in the RGEs  $N_1 = \text{number of } (\mathbf{1}, \mathbf{2}, \mathbf{2}) = 4$  and  $N_{15} = \text{number of } (\mathbf{15}, \mathbf{2}, \mathbf{2}) = 2$  as well as  $\Delta_L = \text{number of } (\overline{\mathbf{10}}, \mathbf{3}, \mathbf{1})_{126} = 0$  and  $\Delta_R = \text{number of } (\mathbf{10}, \mathbf{1}, \mathbf{3})_{126} = 1$  for the  $G_{\text{PS}}$  symmetry breaking.

For the matching conditions at  $M_I$  we took

$$\alpha_{4C}^{-1}(M_I) = \alpha_3^{-1}(M_I) + \frac{1}{12\pi} \quad (3)$$

$$\alpha_{2L}^{-1}(M_I) = \alpha_2^{-1}(M_I) \quad (4)$$

$$\alpha_{2R}^{-1}(M_I) = \frac{5}{3}\alpha_1^{-1}(M_I) - \frac{2}{3}\alpha_3^{-1}(M_I) + \frac{1}{3\pi} \quad (5)$$

while at  $M_U$  the conditions are

$$\alpha_U^{-1}(M_U) = \alpha_{4C}^{-1}(M_U) + \frac{1}{3\pi} \quad (6)$$

$$= \alpha_{2L}^{-1}(M_U) + \frac{1}{2\pi} \quad (7)$$

$$= \alpha_{2R}^{-1}(M_U) + \frac{1}{2\pi} \quad (8)$$

In general the matching conditions between gauge couplings belonging to theories with symmetry groups  $\mathcal{G}_j$  and  $\mathcal{G}_k$  can be written as

$$\alpha_j^{-1}(M_{I/U}) - \frac{1}{12\pi} S_2(\mathcal{G}_j) = \alpha_k^{-1}(M_{I/U}) - \frac{1}{12\pi} S_2(\mathcal{G}_k) \quad (9)$$

where  $S_2(\mathcal{G}_j)$  is the Dynkin index of the adjoint representation of the group  $\mathcal{G}_j$ . Details can be found in [25], the results are given in Table 1. We checked also the RGEs for the

Quantity	$M_I$	$\alpha_1(M_I)$	$\alpha_2(M_I)$
Value	$6.14 \cdot 10^{10} \text{ GeV}$	$(46.00)^{-1}$	$(39.86)^{-1}$
Quantity	$\alpha_3(M_I)$	$\alpha_{2R}(M_I)$	$\alpha_{2L}(M_I)$
Value	$(31.39)^{-1}$	$(55.85)^{-1}$	$(39.86)^{-1}$
Quantity	$\alpha_{4C}(M_I)$	$M_U$	$\alpha_U(M_I)$
Value	$(31.42)^{-1}$	$1.31 \cdot 10^{16} \text{ GeV}$	$(20.08)^{-1}$

Table 1: Symmetry breaking scales and gauge couplings for  $N_1 = 4$  and  $N_{15} = 2$

intermediate symmetry  $G_I = G_{\text{PS}} \otimes D$ , where  $D$  is the discrete  $D$ -parity which requires  $\alpha_{2L} = \alpha_{2R}$  between  $M_U$  and  $M_I$ . In this case however  $M_U(G_{\text{PS}} \otimes D) = 1.16 \cdot 10^{15} \text{ GeV}$ , a value which would lead to a too fast proton decay.

### 3 The mass matrices

The aim of this paper is to present models with large RH and LH mixing angles of the quarks that lead to large mixing of the leptons and to study the predictions of those

<sup>3</sup>Note that due to the quark-lepton symmetry of  $G_{\text{PS}}$  the  $SO(10)$  mass relations are valid also at the scale  $M_I$ .

models for the nucleons decay ratios. To generate the mass matrices we use the method of Harvey, Reiss and Ramond [9] as it was realized in [26] for symmetric mass matrices. However, in order to have large mixing angles one should rather use asymmetric mass matrices. For asymmetric matrices we need to make certain changes in the above scenario and add also the antisymmetric Higgs representation  $\Phi_{\mathbf{120}}$ . Practically speaking, we shall use a global family symmetry  $U(1)_F$  or  $Z_n$  that will dictate the neutrino properties in terms of the observed masses and mixings of the charged fermions. This symmetry will be chosen in such a way that the predictive Fritzsch texture [27] will be realized. However, as it is well known the symmetric version of this texture cannot account for the large top quark mass [28]. As we need anyhow asymmetric mass matrices we shall use the asymmetric Fritzsch texture which is also known under the name “nearest neighbour interaction” model (NNI) [29]. Namely,

$$\mathbf{M} = \begin{pmatrix} 0 & A & 0 \\ B & 0 & C \\ 0 & D & E \end{pmatrix}. \quad (10)$$

In view of the fact that we are actually mainly interested in the predictions for the nucleon decay rates which are not sensitive to the details of  $CP$  violation we will use for simplicity real mass matrices <sup>4</sup>.

The three fermion families and the different Higgs representations in  $\mathbf{16}_i \bar{\Phi}_k \mathbf{16}_j$  transform under the global  $U(1)_F$  as follows

$$\mathbf{16}_j \rightarrow \exp(i\alpha_j\theta)\mathbf{16}_j \quad (11)$$

$$\Phi_k \rightarrow \exp(i\beta_k\theta)\Phi_k \quad (12)$$

The invariance under  $U(1)_F$  requires therefore that the  $\beta_k$  must obey  $\alpha_i + \alpha_j = \beta_k$ . Hence, the fermionic part of the mass matrices has the following quantum numbers:

$$\mathbf{M}_f \sim \begin{pmatrix} \alpha_1 + \alpha_1 & \alpha_1 + \alpha_2 & \alpha_1 + \alpha_3 \\ \alpha_1 + \alpha_2 & \alpha_2 + \alpha_2 & \alpha_2 + \alpha_3 \\ \alpha_1 + \alpha_3 & \alpha_2 + \alpha_3 & \alpha_3 + \alpha_3 \end{pmatrix}. \quad (13)$$

To realize the NNI texture (10) one sees that only Higgs representations with the charges  $\beta = \alpha_1 + \alpha_2, \alpha_2 + \alpha_3$  and  $\alpha_3 + \alpha_3$  can couple to the fermions. Also, we still have the possibility to couple one Higgs representation to two different combinations i.e.  $\alpha_1 + \alpha_2 = 2\alpha_3$ .

Taking all this into account the Yukawa coupling matrices (at energies  $\mu \gtrsim M_I$ ) can have the structure

$$\begin{aligned} \mathbf{Y}_{\mathbf{10}}^{(1)} &= \begin{pmatrix} 0 & x_1 & 0 \\ x_1 & 0 & 0 \\ 0 & 0 & \tilde{x}_1 \end{pmatrix}; & \mathbf{Y}_{\mathbf{126}}^{(1)} &= \begin{pmatrix} 0 & y_1 & 0 \\ y_1 & 0 & 0 \\ 0 & 0 & \tilde{y}_1 \end{pmatrix}; & \mathbf{Y}_{\mathbf{120}}^{(1)} &= \begin{pmatrix} 0 & z_1 & 0 \\ -z_1 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}; \\ \mathbf{Y}_{\mathbf{10}}^{(2)} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & x_2 \\ 0 & x_2 & 0 \end{pmatrix}; & \mathbf{Y}_{\mathbf{126}}^{(2)} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & y_2 \\ 0 & y_2 & 0 \end{pmatrix}; & \mathbf{Y}_{\mathbf{120}}^{(2)} &= \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & z_2 \\ 0 & -z_2 & 0 \end{pmatrix} \end{aligned} \quad (14)$$

On top of that, we need at least one  $\Phi_{\mathbf{126}}$  with a large VEV in  $(\mathbf{10}, \mathbf{1}, \mathbf{3})$  to break the  $SO(10)$  gauge symmetry at  $M_I$ . This VEV will generate also the RH neutrinos masses

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<sup>4</sup>This can help to solve the strong  $CP$  problem while the observed  $CP$  violation in the  $K$ -decay can come from a different origin than the CKM matrix.

$M_R \sim M_I$ . Actually, as is discussed in App. I we will use only one  $\Phi_{\mathbf{126}}$  in addition to two  $\Phi_{\mathbf{10}}$  and two  $\Phi_{\mathbf{120}}$  to generate the asymmetrical mass matrices. The detailed fits required four VEVs in the direction  $(\mathbf{1}, \mathbf{2}, \mathbf{2})$  and two in that of  $(\mathbf{15}, \mathbf{2}, \mathbf{2})$ . Those give also the right unification as discussed before.

In terms of the notation and discussion of App. I we obtained the following expressions for the mass matrix elements in terms of the VEVs and Yukawa couplings:

$$(\mathbf{M}_d)_{12} = x_1 v_d^{(1)} + y_1 \omega_d^{(1)} + z_1 \tilde{v}_d^{(1)} \quad (15)$$

$$(\mathbf{M}_d)_{21} = x_1 v_d^{(1)} + y_1 \omega_d^{(1)} - z_1 \tilde{v}_d^{(1)} \quad (16)$$

$$(\mathbf{M}_d)_{23} = x_2 v_d^{(2)} + z_2 (\tilde{v}_d^{(2)} + \tilde{\omega}_d^{(2)}) \quad (17)$$

$$(\mathbf{M}_d)_{32} = x_2 v_d^{(2)} - z_2 (\tilde{v}_d^{(2)} + \tilde{\omega}_d^{(2)}) \quad (18)$$

$$\begin{aligned} (\mathbf{M}_d)_{33} &= \tilde{x}_1 v_d^{(1)} + \tilde{y}_1 \omega_d^{(1)} \\ &= \left( \frac{\tilde{x}_1}{x_1} \right) x_1 v_d^{(1)} + \left( \frac{\tilde{y}_1}{y_1} \right) y_1 \omega_d^{(1)} \end{aligned} \quad (19)$$

$$\begin{aligned} (\mathbf{M}_e)_{12} &= x_1 v_d^{(1)} - 3 y_1 \omega_d^{(1)} + z_1 \tilde{v}_d^{(1)} \\ &= (\mathbf{M}_d)_{12} - 4 y_1 \omega_d^{(1)} \end{aligned} \quad (20)$$

$$\begin{aligned} (\mathbf{M}_e)_{21} &= x_1 v_d^{(1)} - 3 y_1 \omega_d^{(1)} - z_1 \tilde{v}_d^{(1)} \\ &= (\mathbf{M}_d)_{21} - 4 y_1 \omega_d^{(1)} \end{aligned} \quad (21)$$

$$\begin{aligned} (\mathbf{M}_e)_{23} &= x_2 v_d^{(2)} + z_2 (\tilde{v}_d^{(2)} - 3 \tilde{\omega}_d^{(2)}) \\ &= (\mathbf{M}_d)_{23} - 4 z_2 \tilde{\omega}_d^{(2)} \end{aligned} \quad (22)$$

$$\begin{aligned} (\mathbf{M}_e)_{32} &= x_2 v_d^{(2)} - z_2 (\tilde{v}_d^{(2)} - 3 \tilde{\omega}_d^{(2)}) \\ &= (\mathbf{M}_d)_{32} + 4 z_2 \tilde{\omega}_d^{(2)} \end{aligned} \quad (23)$$

$$\begin{aligned} (\mathbf{M}_e)_{33} &= \tilde{x}_1 v_d^{(1)} - 3 \tilde{y}_1 \omega_d^{(1)} \\ &= (\mathbf{M}_d)_{33} - 4 \left( \frac{\tilde{y}_1}{y_1} \right) y_1 \omega_d^{(1)} \end{aligned} \quad (24)$$

$$(\mathbf{M}_u)_{12} = x_1 v_u^{(1)} + y_1 \omega_u^{(1)} + z_1 \tilde{v}_u^{(1)} \quad (25)$$

$$(\mathbf{M}_u)_{21} = x_1 v_u^{(1)} + y_1 \omega_u^{(1)} - z_1 \tilde{v}_u^{(1)} \quad (26)$$

$$(\mathbf{M}_u)_{23} = x_2 v_u^{(2)} + z_2 (\tilde{v}_u^{(2)} + \tilde{\omega}_u^{(2)}) \quad (27)$$

$$(\mathbf{M}_u)_{32} = x_2 v_u^{(2)} - z_2 (\tilde{v}_u^{(2)} + \tilde{\omega}_u^{(2)}) \quad (28)$$

$$\begin{aligned} (\mathbf{M}_u)_{33} &= \tilde{x}_1 v_u^{(1)} + \tilde{y}_1 \omega_u^{(1)} \\ &= \left( \frac{\tilde{x}_1}{x_1} \right) x_1 v_u^{(1)} + \left( \frac{\tilde{y}_1}{y_1} \right) y_1 \omega_u^{(1)} \end{aligned} \quad (29)$$

$$\begin{aligned} (\mathbf{M}_\nu^{(\text{Dir})})_{12} &= x_1 v_u^{(1)} - 3 y_1 \omega_u^{(1)} + z_1 \tilde{v}_u^{(1)} \\ &= (\mathbf{M}_u)_{12} - 4 y_1 \omega_u^{(1)} \end{aligned} \quad (30)$$

$$\begin{aligned} (\mathbf{M}_\nu^{(\text{Dir})})_{21} &= x_1 v_u^{(1)} - 3 y_1 \omega_u^{(1)} - z_1 \tilde{v}_u^{(1)} \\ &= (\mathbf{M}_u)_{21} - 4 y_1 \omega_u^{(1)} \end{aligned} \quad (31)$$

$$\begin{aligned} (\mathbf{M}_\nu^{(\text{Dir})})_{23} &= x_2 v_u^{(2)} + z_2 (\tilde{v}_u^{(2)} - 3 \tilde{\omega}_u^{(2)}) \\ &= (\mathbf{M}_u)_{23} - 4 z_2 \tilde{\omega}_u^{(2)} \end{aligned} \quad (32)$$

$$\begin{aligned} (\mathbf{M}_\nu^{(\text{Dir})})_{32} &= x_2 v_u^{(2)} - z_2 (\tilde{v}_u^{(2)} - 3 \tilde{\omega}_u^{(2)}) \\ &= (\mathbf{M}_u)_{32} + 4 z_2 \tilde{\omega}_u^{(2)} \end{aligned} \quad (33)$$

$$\begin{aligned}
(\mathbf{M}_\nu^{(\text{Dir})})_{33} &= \tilde{x}_1 v_u^{(1)} - 3 \tilde{y}_1 \omega_u^{(1)} \\
&= (\mathbf{M}_u)_{33} - 4 \left( \frac{\tilde{y}_1}{y_1} \right) y_1 \omega_u^{(1)}
\end{aligned} \tag{34}$$

All those elements are obtained from 14 independent Higgs parameters which are products of VEVs and Yukawa couplings. They also fix the matrix elements of the RH neutrino Majorana mass matrix (in terms of the VEV of  $(\mathbf{10}, \mathbf{1}, \mathbf{3})$  that breaks  $G_{\text{PS}} \rightarrow G_{\text{SM}}$ ), with  $M_R \sim M_I$  being a quasi-free parameter:

$$\mathbf{M}_{\nu_R}^{(\text{Maj})} = M_R \begin{pmatrix} 0 & y_1 & 0 \\ y_1 & 0 & 0 \\ 0 & 0 & \tilde{y}_1 \end{pmatrix} = y_1 M_R \begin{pmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & \tilde{y}_1/y_1 \end{pmatrix} \tag{35}$$

By diagonalizing the charged fermion mass matrices

$$\begin{aligned}
\mathbf{U}_L^\dagger \mathbf{M}_u \mathbf{U}_R &= \mathbf{M}_u^{(D)}, & \mathbf{D}_L^\dagger \mathbf{M}_d \mathbf{D}_R &= \mathbf{M}_d^{(D)}, \\
\mathbf{E}_L^\dagger \mathbf{M}_e \mathbf{E}_R &= \mathbf{M}_e^{(D)}, & \mathbf{U}_L^\dagger \mathbf{D}_L &= \mathbf{V}_{\text{CKM}},
\end{aligned} \tag{36}$$

and the light see-saw neutrino mass matrix

$$\mathbf{M}_\nu^{\text{light}} \simeq -\mathbf{M}_\nu^{(\text{Dir})} (\mathbf{M}_{\nu_R}^{(\text{Maj})})^{-1} (\mathbf{M}_\nu^{(\text{Dir})})^T \tag{37}$$

we obtained all masses of the charged fermions and those of the three light neutrinos as well as the mixing matrices  $\mathbf{U}_{L,R}$ ,  $\mathbf{D}_{L,R}$ ,  $\mathbf{E}_{L,R}$  and  $\mathbf{N}_\nu$  <sup>5</sup>.

Neglecting phases (as explained before) we have 12 masses and 21 mixing parameters. All those fix the proton decay rates and other GUT scale effects (like baryon asymmetry induced by leptogenesis [18]). However, in the framework of the SM (with massive neutrinos) the only observable mixing matrices would be  $\mathbf{V}_{\text{CKM}} = \mathbf{U}_L^\dagger \mathbf{D}_L$  and  $\mathbf{U} = \mathbf{E}_L^\dagger \mathbf{N}_\nu$ . This gives 18 mixing observables <sup>6</sup>.

## 4 Numerical solutions for the mass matrices

The first step will be to calculate numerically the mass and mixing matrices in (36). This is done using the  $SO(10)$  relations (15-34) for the mass matrices at  $\mu = M_I = 6.14 \cdot 10^{10}$  GeV. On the RHS of (36) we use the calculated masses of the charged fermions (see Table 2) and the (real) CKM matrix at  $\mu = M_I$ . Those values are obtained using the RGEs as is described in detail in [25]. We then run the mass matrices from  $M_I$  to  $M_Z$ , diagonalize them there and compare the obtained masses and mixings with their experimentally observed values.

Now, without using the neutrino sector, only 13 of our 14 parameters are independent because only the combination  $z_2(\tilde{v}_u^{(2)} + \tilde{\omega}_u^{(2)})$  appears in the charged fermion equations. We have in (36) 30 nonlinear equations. On the other hand each of the (real) 6 mixing matrices is parametrized by 3 angles, so altogether there are 18 mixing angles. With the 13 Higgs parameters we have 31 “unknowns”. One of them must therefore be “given” to be able to search for solutions numerically. For the given quantity we use the ratio  $\tilde{y}_1/y_1$  which determines the RH neutrino mass matrix (35) except for a global factor and we vary

<sup>5</sup>There is only one neutrino mixing matrix as  $\mathbf{M}_\nu^{\text{light}}$  is always symmetric.

<sup>6</sup>Note however, given the mass matrices we have definite predictions for the proton decay, the baryon asymmetry and RH currents which can be measured in principle by future experiments.

Mass	$m_u(M_I)$	$m_d(M_I)$	$m_s(M_I)$
Value	1.16 MeV	2.38 MeV	47.4 MeV
Mass	$m_c(M_I)$	$m_b(M_I)$	$m_t(M_I)$
Value	337.6 MeV	1360 MeV	101.2 GeV
Mass	$m_e(M_I)$	$m_\mu(M_I)$	$m_\tau(M_I)$
Value	513 keV	108.14 MeV	1838.3 MeV

Table 2: Fermion masses at  $M_I$

Solution	MSW effect	$\tilde{y}_1/y_1$	$z_2\tilde{\omega}_u^{(2)}$ (MeV)
Model 1	large mixing	19	-3450
Model 2a	small mixing	24	-9175
Model 2b	small mixing	-18	-9925

Table 3: Values of  $\tilde{y}_1/y_1$  and  $z_2\tilde{\omega}_u^{(2)}$  for the three representative solutions

its value between  $1 \leq |\tilde{y}_1/y_1| \leq 1000$ . By studying the equations in detail one can see that, once the value of  $\tilde{y}_1/y_1$  is given, the neutrino sector of our model is uniquely fixed up to the two parameters  $M_R \sim M_I$  and  $z_2\tilde{\omega}_u^{(2)}$ . We therefore look if there are solutions of the model for reasonable values of  $\tilde{y}_1/y_1$ ,  $M_R$  and  $z_2\tilde{\omega}_u^{(2)}$  which predict neutrino properties lying in the range allowed by oscillation experiments [2, 3, 20, 21]:

$$|(\mathbf{U})_{13}| \leq 0.05 \quad (38)$$

$$0.49 \leq |(\mathbf{U})_{23}| \leq 0.71 \quad (39)$$

$$0.03 \leq |(\mathbf{U})_{12}| \leq 0.05 \quad (\text{small angle MSW})$$

$$\text{or } 0.35 \leq |(\mathbf{U})_{12}| \leq 0.49 \quad (\text{large angle MSW}) \quad (40)$$

$$50 \leq \Delta m_{\text{atm}}^2 / \Delta m_{\text{sun}}^2 \equiv (m_{\nu_3}^2 - m_{\nu_2}^2) / (m_{\nu_2}^2 - m_{\nu_1}^2) \leq 1000 \quad (41)$$

The value of  $y_1 M_R$  will be fixed at the end to give the exact absolute scale of the neutrino masses.

Given these three parameters the model predicts the three neutrino masses and three lepton mixing angles in  $\mathbf{U}$ . We found two regions in the  $\tilde{y}_1/y_1$ - $z_2\tilde{\omega}_u^{(2)}$  parameter space which obey the atmospheric neutrino requirements together with small angle MSW [30] explanation for the solar neutrino puzzle (models 2a,b) and one with the large angle MSW (model 1) as can be seen in the Figures 1, 2 and 3. For each region we fixed a representative solution (see Table 3) and used it to calculate the corresponding neutrino properties. The explicit results for the neutrinos are given in Table 4.

In this way the explicit LH and RH mixing matrices are also fixed. The corresponding mixing angles are given in Table 5. They are used to calculate the branching ratios of the nucleon decays.



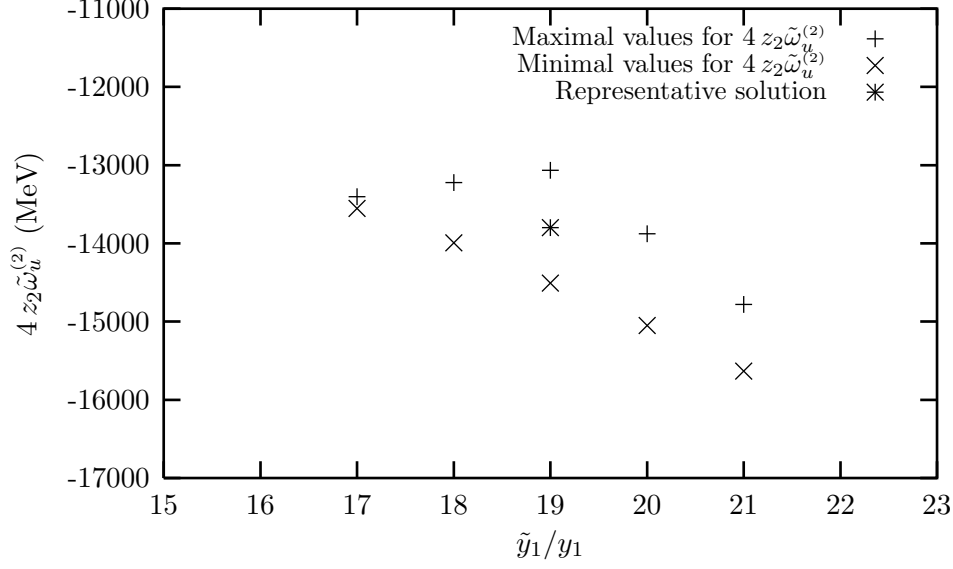


Figure 1: Solution 1 (large mixing MSW) in the  $\tilde{y}_1/y_1$ - $z_2 \tilde{\omega}_u^{(2)}$  parameter space

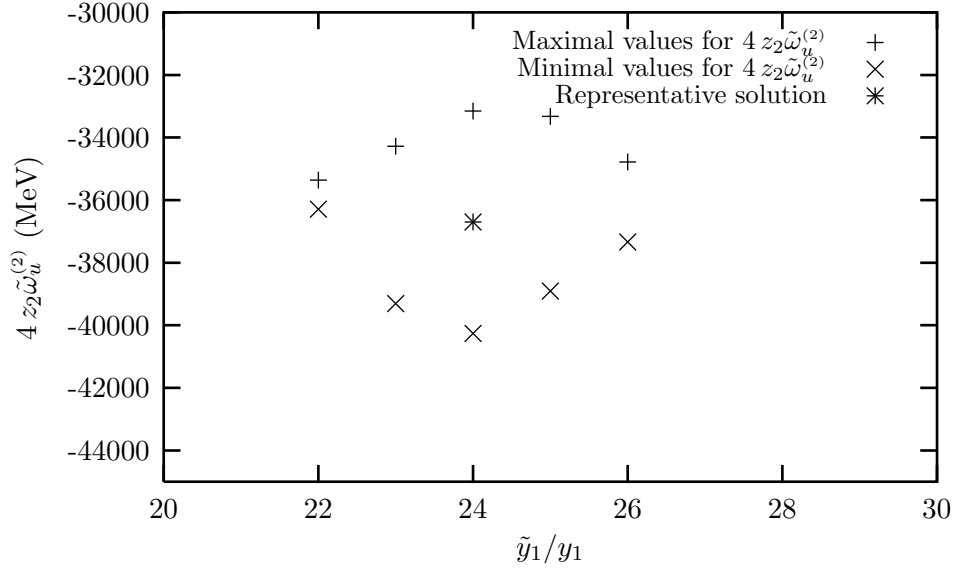


Figure 2: Solution 2a (small mixing MSW) in the  $\tilde{y}_1/y_1$ - $z_2 \tilde{\omega}_u^{(2)}$  parameter space

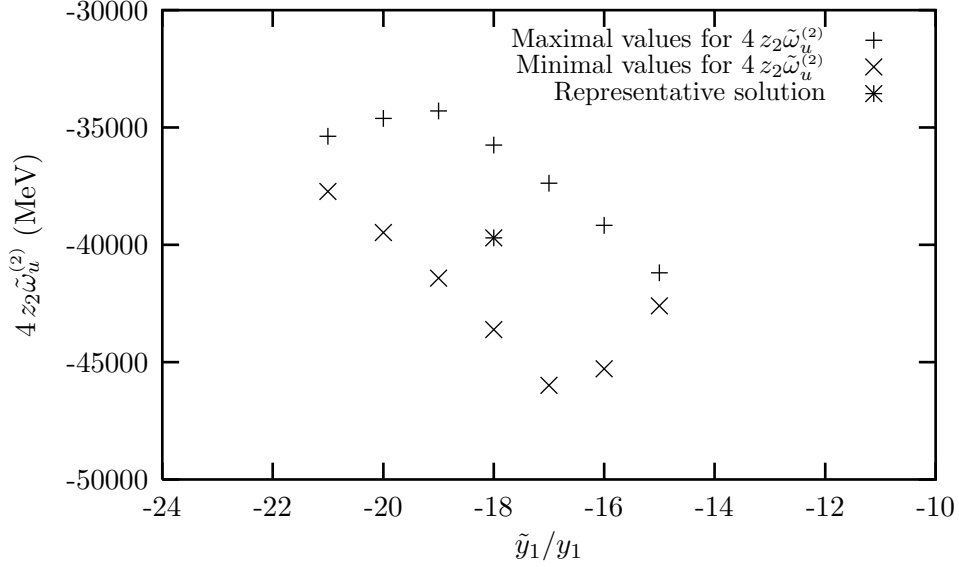


Figure 3: Solution 2b (small mixing MSW) in the  $\tilde{y}_1/y_1$ - $z_2\tilde{\omega}_u^{(2)}$  parameter space

## 5 Calculation of the nucleon decay rates

The partial decay rate for a given process nucleon  $\rightarrow$  meson + antilepton is expressed as follows:

$$\Gamma_j = \frac{1}{16\pi} m_{\text{nuc}}^2 \rho_j |S|^2 |\mathcal{A}|^2 \left( |\mathcal{A}_L|^2 \sum_l |A_l \mathcal{M}_l|^2 + |\mathcal{A}_R|^2 \sum_r |A_r \mathcal{M}_r|^2 \right), \quad (42)$$

where  $\mathcal{M}_l$  and  $\mathcal{M}_r$  are the hadronic transition matrix elements for the relevant decay process.  $l$  and  $r$  denote the chirality of the corresponding antilepton.  $A_l$  and  $A_r$  are the relevant coefficients of the effective Lagrangian (67) given in App. II.  $\mathcal{A}$ ,  $\mathcal{A}_L$  and  $\mathcal{A}_R$  are factors which result from the renormalization of the four fermion operators as follows:

$$\mathcal{A}_L = \left( \frac{\alpha_1(M_Z)}{\alpha_1(M_I)} \right)^{-\frac{23}{82}} \quad (43)$$

$$\mathcal{A}_R = \left( \frac{\alpha_1(M_Z)}{\alpha_1(M_I)} \right)^{-\frac{11}{82}} \quad (44)$$

$$\begin{aligned} \mathcal{A} &= \left( \frac{\alpha_{4C}(M_I)}{\alpha_{4C}(M_U)} \right)^{-\frac{5}{8}} \left( \frac{\alpha_{2L}(M_I)}{\alpha_{2L}(M_U)} \right)^{-\frac{27}{100}} \left( \frac{\alpha_{2R}(M_I)}{\alpha_{2R}(M_U)} \right)^{-\frac{3}{20}} \left( \frac{\alpha_2(M_Z)}{\alpha_2(M_I)} \right)^{\frac{27}{38}} \\ &\quad \cdot \left( \frac{\alpha_3(M_Z)}{\alpha_3(M_I)} \right)^{\frac{2}{7}} \left( \frac{\alpha_3(m_b)}{\alpha_3(M_Z)} \right)^{\frac{6}{23}} \left( \frac{\alpha_3(m_c)}{\alpha_3(m_b)} \right)^{\frac{6}{25}} \left( \frac{\alpha_3(1 \text{ GeV})}{\alpha_3(m_c)} \right)^{\frac{2}{9}} \end{aligned} \quad (45)$$

Using then [31]

$$\alpha_3(1 \text{ GeV}) = 0.544, \quad \alpha_3(m_c) = 0.412, \quad \alpha_3(m_b) = 0.226 \quad (46)$$

one obtains

$$|\mathcal{A}_L|^2 = 1.155, \quad |\mathcal{A}_R|^2 = 1.071, \quad |\mathcal{A}|^2 = 23.59. \quad (47)$$

Parameter	Value in Model 1	Value in Model 2a	Value in Model 2b
$(y_1 M_R/M_I) \cdot m_{\nu_1}$	$-0.0245 \text{ eV}$	$-8.73 \cdot 10^{-4} \text{ eV}$	$1.16 \cdot 10^{-3} \text{ eV}$
$(y_1 M_R/M_I) \cdot m_{\nu_2}$	$0.0876 \text{ eV}$	$0.355 \text{ eV}$	$-0.467 \text{ eV}$
$(y_1 M_R/M_I) \cdot m_{\nu_3}$	$-2.402 \text{ eV}$	$-3.031 \text{ eV}$	$4.365 \text{ eV}$
$\theta_{12}^{(\nu)}$	$-0.487$	$-0.050$	$0.051$
$\theta_{23}^{(\nu)}$	$0.205$	$0.506$	$0.496$
$\theta_{31}^{(\nu)}$	$0.004$	$0.003$	$-0.003$
$m_{\nu_2}/m_{\nu_1}$	$-3.57$	$-406.3$	$-401.2$
$m_{\nu_3}/m_{\nu_2}$	$-27.43$	$-8.54$	$-9.35$
$\left(\frac{m_{\nu_3}^2 - m_{\nu_2}^2}{m_{\nu_2}^2 - m_{\nu_1}^2}\right)$	$815.4$	$71.9$	$86.4$

Table 4: Masses and mixing angles of the light neutrinos at  $M_Z$

$|S|^2 = \langle \Psi_{\text{Nuc}}^s(\vec{r}_1, \vec{r}_2, \vec{r}_3) | \delta(\vec{r}_1 - \vec{r}_2) | \Psi_{\text{Nuc}}^s(\vec{r}_1, \vec{r}_2, \vec{r}_3) \rangle$  is the probability to find two valence quarks of the nucleon at one point in space. We used here the value  $|S|^2 = 0.012 \text{ GeV}^3$  given in [32].  $\rho_j \equiv (1 - \chi_j^2)(1 - \chi_j^4)$  with  $\chi_j = m_{\text{Meson}}/m_{\text{Nuc}}$  is an  $SU(6)$  spin-flavour symmetry breaking phase space factor.

The resulting branching ratios are given in the Tables 6 and 8. Our model also gives the total decay rates. The predicted rates in the different models are given in the Tables 7 and 9. Note however that these predictions have a relatively large uncertainty. This was estimated by Langacker [33] to be

$$\Delta\tau_{p \rightarrow e^+ \pi^0} = 10^{\pm 0.7 \pm 1.0^{+0.5}_{-3.0}} \text{ yrs.} \quad (48)$$

Taking this into account our predicted representative rates have therefore a chance to be observed by SuperKamiokande [34] and ICARUS [35].

Yet the essential predictions of the model are actually the branching ratios. Comparing our branching ratios with those of the conventional  $SO(10)$  (i.e. without large mixings) one sees clearly the suppression of the  $p, n \rightarrow e^+ X$  channels relative to the channels  $p, n \rightarrow \mu^+ X, \nu^c X$ . In particular  $p, n \rightarrow \mu^+ \pi, \nu^c K$  are prominent. Note that in SUSY GUTs [12] the dominant decays are into final states involving  $K$  mesons<sup>7</sup>. The special properties of our model are clearly reflected in the comparison of ratios as is done in Table 10.

## 6 Conclusions

We presented an  $SO(10)$  GUT with a global  $U(1)_F$  family symmetry that is consistent with all experimental observations. The special thing about this model is that it involves large mixing angles for the quarks, in contrast with the conventional expectations. The large mixing in the lepton (neutrino) sector results therefore naturally.

Large rotations have considerable consequences for observables outside the Standard Model. In particular calculations of the nucleon decay rates require the knowledge of all mixing angles. We found rates which are obviously different from the conventional GUTs. Our model is however only one simple example for the effects of large mixing angles. We are

<sup>7</sup>This is also the case for non-SUSY GUTs with maximal RH mixings [36].

Parameter	Value in model 1	Value in model 2a	Value in model 2b
$\theta_{L12}^{(u)}$	-0.627	-0.540	0.275
$\theta_{L23}^{(u)}$	-0.055	-0.056	-0.075
$\theta_{L31}^{(u)}$	0.000	0.000	0.000
$\theta_{R12}^{(u)}$	0.005	0.006	-0.012
$\theta_{R23}^{(u)}$	0.049	0.051	0.043
$\theta_{R31}^{(u)}$	0.000	0.000	0.000
$\theta_{L12}^{(d)}$	-0.404	-0.317	0.498
$\theta_{L23}^{(d)}$	-0.025	-0.024	-0.041
$\theta_{L31}^{(d)}$	-0.018	-0.016	0.012
$\theta_{R12}^{(d)}$	0.117	0.152	-0.092
$\theta_{R23}^{(d)}$	0.979	1.011	0.679
$\theta_{R31}^{(d)}$	0.000	0.000	0.000
$\theta_{L12}^{(e)}$	-0.018	-0.015	0.015
$\theta_{L23}^{(e)}$	0.733	-0.060	-0.089
$\theta_{L31}^{(e)}$	0.000	-0.001	0.001
$\theta_{R12}^{(e)}$	0.262	0.314	-0.301
$\theta_{R23}^{(e)}$	-0.063	0.747	0.561
$\theta_{R31}^{(e)}$	0.014	-0.001	0.002
$\theta_{12}^{(\nu)}$	-0.487	-0.050	0.051
$\theta_{23}^{(\nu)}$	0.205	0.506	0.496
$\theta_{31}^{(\nu)}$	0.004	0.003	-0.003

Table 5: Mixing angles in the three different models

studying now a SUSY GUT with all possible phases and its consequences also for other GUT observables or RH currents. The hope is that by restricting the values of the mixing angles, we will be able to reduce the large freedom in the present models for fermionic masses.

## Acknowledgments

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Decay channel of the proton	Rates in % (no mixing)	Rates in % in model 1	Rates in % in model 2a	Rates in % in model 2b
$p \rightarrow e^+ \pi^0$	33.6	21.4	25.1	27.8
$p \rightarrow e^+ K^0$	—	3.1	2.6	4.5
$p \rightarrow e^+ \eta$	1.2	0.8	0.9	1.0
$p \rightarrow \mu^+ \pi^0$	—	8.5	5.7	5.6
$p \rightarrow \mu^+ K^0$	5.8	2.6	0.9	1.8
$p \rightarrow \mu^+ \eta$	—	0.3	0.2	0.2
$p \rightarrow e^+ \rho^0$	5.1	3.3	3.8	4.2
$p \rightarrow e^+ \omega$	16.9	10.8	12.7	14.0
$p \rightarrow e^+ K^{*0}$	—	0.0	0.0	0.0
$p \rightarrow \mu^+ \rho^0$	—	1.3	0.9	0.8
$p \rightarrow \mu^+ \omega$	—	4.3	2.9	2.8
$p \rightarrow \nu_e^C \pi^+$	32.3	25.6	35.6	27.7
$p \rightarrow \nu_e^C K^+$	—	2.0	2.0	4.1
$p \rightarrow \nu_\mu^C \pi^+$	—	8.9	0.5	0.3
$p \rightarrow \nu_\mu^C K^+$	0.1	1.3	0.2	0.3
$p \rightarrow \nu_e^C \rho^+$	4.9	3.9	5.4	4.2
$p \rightarrow \nu_e^C K^{*+}$	—	0.4	0.3	0.6
$p \rightarrow \nu_\mu^C \rho^+$	—	1.4	0.1	0.0
$p \rightarrow \nu_\mu^C K^{*+}$	0.1	0.0	0.0	0.0
$p \rightarrow \nu_\tau^C \pi^+$	—	0.1	0.1	0.0
$p \rightarrow \nu_\tau^C K^+$	—	0.1	0.1	0.0
$p \rightarrow \nu_\tau^C \rho^+$	—	0.0	0.0	0.0
$p \rightarrow \nu_\tau^C K^{*+}$	—	0.0	0.0	0.0
$p \rightarrow e^+ X^0$	56.8	39.4	45.1	51.5
$p \rightarrow \mu^+ X^0$	5.8	17.0	10.6	11.2
$p \rightarrow \nu^C X^+$	37.4	43.7	44.3	37.2

Table 6: Partial decay rates  $\Gamma_i/\Gamma$  of the proton for the case of vanishing fermionic mixing in comparison with the rates obtained in our solutions

Quantity	Value in model 1	Value in model 2a	Value in model 2b
$\Gamma_p$	$2.54 \cdot 10^{-35} \text{ yr}^{-1}$	$2.57 \cdot 10^{-35} \text{ yr}^{-1}$	$2.83 \cdot 10^{-35} \text{ yr}^{-1}$
$\tau_p$	$3.94 \cdot 10^{34} \text{ yr}$	$3.89 \cdot 10^{34} \text{ yr}$	$3.53 \cdot 10^{34} \text{ yr}$

Table 7: Total decay rate and lifetime of the proton for the representative solutions

Decay channel of the neutron	Rates in % (no mixing)	Rates in % in model 1	Rates in % in model 2a	Rates in % in model 2b
$n \rightarrow e^+ \pi^-$	62.9	40.1	46.2	49.9
$n \rightarrow \mu^+ \pi^-$	—	15.8	10.4	10.0
$n \rightarrow e^+ \rho^-$	9.7	6.2	7.1	7.7
$n \rightarrow \mu^+ \rho^-$	—	2.4	1.6	1.5
$n \rightarrow \nu_e^C \pi^0$	15.1	12.0	16.4	12.5
$n \rightarrow \nu_e^C K^0$	—	6.8	4.9	8.2
$n \rightarrow \nu_e^C \eta$	0.6	0.4	0.6	0.5
$n \rightarrow \nu_\mu^C \pi^0$	—	4.2	0.2	0.1
$n \rightarrow \nu_\mu^C K^0$	1.7	0.1	1.0	0.9
$n \rightarrow \nu_\mu^C \eta$	—	0.2	0.0	0.0
$n \rightarrow \nu_e^C \rho^0$	2.3	1.8	2.5	1.9
$n \rightarrow \nu_e^C \omega$	7.7	6.1	8.4	6.4
$n \rightarrow \nu_e^C K^{*0}$	—	0.2	0.2	0.4
$n \rightarrow \nu_\mu^C \rho^0$	—	0.6	0.0	0.0
$n \rightarrow \nu_\mu^C \omega$	—	2.1	0.1	0.1
$n \rightarrow \nu_\mu^C K^{*0}$	0.0	0.1	0.0	0.0
$n \rightarrow \nu_\tau^C \pi^0$	—	0.0	0.0	0.0
$n \rightarrow \nu_\tau^C K^0$	—	0.7	0.4	0.0
$n \rightarrow \nu_\tau^C \eta$	—	0.0	0.0	0.0
$n \rightarrow \nu_\tau^C \rho^0$	—	0.0	0.0	0.0
$n \rightarrow \nu_\tau^C \omega$	—	0.0	0.0	0.0
$n \rightarrow \nu_\tau^C K^{*0}$	—	0.0	0.0	0.0
$n \rightarrow e^+ X^-$	72.6	46.3	53.3	57.6
$n \rightarrow \mu^+ X^-$	—	18.2	12.0	11.5
$n \rightarrow \nu^C X^0$	27.4	35.3	34.7	31.0

Table 8: Partial decay rates  $\Gamma_i/\Gamma$  of the bound neutron for the case of vanishing fermionic mixing in comparison with the rates obtained in our solutions

Quantity	Value in model 1	Value in model 2a	Value in model 2b
$\Gamma_n$	$2.72 \cdot 10^{-35} \text{ yr}^{-1}$	$2.80 \cdot 10^{-35} \text{ yr}^{-1}$	$3.14 \cdot 10^{-35} \text{ yr}^{-1}$
$\tau_n$	$3.68 \cdot 10^{34} \text{ yr}$	$3.57 \cdot 10^{34} \text{ yr}$	$3.18 \cdot 10^{34} \text{ yr}$

Table 9: Total decay rate and lifetime of the bound neutron for the representative solutions

Ratio	No mixing	Model 1	Model 2a	Model 2b
$\frac{\Gamma(p \rightarrow e^+ K^0)}{\Gamma(p \rightarrow e^+ \pi^0)}$	0	0.145	0.104	0.162
$\frac{\Gamma(p \rightarrow \mu^+ \pi^0)}{\Gamma(p \rightarrow \mu^+ K^0)}$	0	3.27	6.33	3.11
$\frac{\Gamma(p \rightarrow \nu^c K^+)}{\Gamma(p \rightarrow \nu^c \pi^+)}$	0.003	0.098	0.064	0.157
$\frac{\Gamma(p \rightarrow e^+ \pi^0)}{\Gamma(p \rightarrow \nu^c \pi^+)}$	1.040	0.618	0.693	0.993
$\frac{\Gamma(n \rightarrow \mu^+ \pi^-)}{\Gamma(n \rightarrow e^+ \pi^-)}$	0	0.394	0.225	0.200
$\frac{\Gamma(n \rightarrow \mu^+ \rho^-)}{\Gamma(n \rightarrow e^+ \rho^-)}$	0	0.387	0.225	0.195
$\frac{\Gamma(n \rightarrow \nu^c K^0)}{\Gamma(n \rightarrow \nu^c \pi^0)}$	0.113	0.469	0.347	0.722
$\frac{\Gamma(n \rightarrow e^+ \pi^-)}{\Gamma(n \rightarrow \nu^c \pi^0)}$	4.16	2.48	2.78	3.96

Table 10: Ratios of some partial nucleon decay rates

## Appendix I: Fermionic masses in $SO(10)$ GUTs

Dirac masses for the charged fermions are of the form  $m(\Psi_L^c)^T C \Psi_L$  and therefore transform under  $SO(10)$  as follows:

$$\mathbf{16} \otimes \mathbf{16} = (\mathbf{10} \oplus \mathbf{126})_{\text{symm}} \oplus \mathbf{120}_{\text{antisymm}} \quad (49)$$

The  $SO(10)$  Higgs representations which can give masses to the fermions have the representation content

$$\mathbf{10} \longrightarrow (\mathbf{1}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{6}, \mathbf{1}, \mathbf{1}) \quad (50)$$

$$\mathbf{120} \longrightarrow (\mathbf{1}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{15}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{6}, \mathbf{3}, \mathbf{1}) \oplus (\mathbf{6}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{10}, \mathbf{1}, \mathbf{1}) \oplus (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{1}) \quad (51)$$

$$\mathbf{126} \longrightarrow (\mathbf{15}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{10}, \mathbf{3}, \mathbf{1}) \oplus (\overline{\mathbf{10}}, \mathbf{1}, \mathbf{3}) \oplus (\mathbf{6}, \mathbf{1}, \mathbf{1}) \quad (52)$$

under  $G_{\text{PS}} = SU(4)_C \otimes SU(2)_L \otimes SU(2)_R$ , while for the fermions in the  $\mathbf{16}$

$$\mathbf{16} \longrightarrow (\mathbf{4}, \mathbf{2}, \mathbf{1}) \oplus (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \quad (53)$$

Hence the Dirac mass terms transform as follows

$$(\mathbf{4}, \mathbf{2}, \mathbf{1}) \otimes (\overline{\mathbf{4}}, \mathbf{1}, \mathbf{2}) = (\mathbf{15}, \mathbf{2}, \mathbf{2}) \oplus (\mathbf{1}, \mathbf{2}, \mathbf{2}) \quad (54)$$

The different Yukawa couplings and the corresponding VEVs of  $\Phi_{\mathbf{10}}$ ,  $\Phi_{\mathbf{120}}$  and  $\Phi_{\mathbf{126}}$  are given in Table 11.

Taking now the Clebsch-Gordan coefficients into account the mass matrices get the following contributions [38]

$$\mathbf{M}_d = v_d \mathbf{Y}_{\mathbf{10}} + \omega_d \mathbf{Y}_{\mathbf{126}} + (\tilde{v}_d + \tilde{\omega}_d) \mathbf{Y}_{\mathbf{120}} \quad (55)$$

$$\mathbf{M}_e = v_d \mathbf{Y}_{\mathbf{10}} - 3\omega_d \mathbf{Y}_{\mathbf{126}} + (\tilde{v}_d - 3\tilde{\omega}_d) \mathbf{Y}_{\mathbf{120}} \quad (56)$$

$$\mathbf{M}_u = v_u \mathbf{Y}_{\mathbf{10}} + \omega_u \mathbf{Y}_{\mathbf{126}} + (\tilde{v}_u + \tilde{\omega}_u) \mathbf{Y}_{\mathbf{120}} \quad (57)$$

$$\mathbf{M}_\nu^{(\text{Dir})} = v_u \mathbf{Y}_{\mathbf{10}} - 3\omega_u \mathbf{Y}_{\mathbf{126}} + (\tilde{v}_u - 3\tilde{\omega}_u) \mathbf{Y}_{\mathbf{120}} \quad (58)$$

Higgs representation	$(\mathbf{1}, \mathbf{2}, \mathbf{2})_{10}^{(i)}$	$(\mathbf{1}, \mathbf{2}, \mathbf{2})_{120}^{(j)}$	$(\mathbf{15}, \mathbf{2}, \mathbf{2})_{120}^{(k)}$	$(\mathbf{15}, \mathbf{2}, \mathbf{2})_{126}^{(l)}$
Yukawa coupling matrix	$\mathbf{Y}_{10}^{(i)}$	$\mathbf{Y}_{120}^{(j)}$	$\mathbf{Y}_{120}^{(k)}$	$\mathbf{Y}_{126}^{(l)}$
Vacuum expectation values	$v_u^{(i)}, v_d^{(i)}$	$\tilde{v}_u^{(j)}, \tilde{v}_d^{(j)}$	$\tilde{\omega}_u^{(k)}, \tilde{\omega}_d^{(k)}$	$\omega_u^{(l)}, \omega_d^{(l)}$

Table 11: Higgs couplings and vacuum expectation values in  $SO(10)$  GUTs

The RH neutrinos can acquire also Majorana masses in term of the  $(\mathbf{10}, \mathbf{1}, \mathbf{3})$  component of  $\Phi_{126}$ :

$$(\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \otimes (\bar{\mathbf{4}}, \mathbf{1}, \mathbf{2}) \otimes (\mathbf{10}, \mathbf{1}, \mathbf{3}) = (\mathbf{1}, \mathbf{1}, \mathbf{1}) \oplus \dots \quad (59)$$

$$\mathbf{M}_{\nu R}^{(\text{Maj})} = M_R \mathbf{Y}_{126} \sim M_I \mathbf{Y}_{126} \quad (60)$$

The corresponding VEV  $M_I$  is responsible for the symmetry breaking step  $G_{\text{PS}} \rightarrow G_{\text{SM}}$ . The neutrinos will have therefore the  $6 \times 6$  mass matrix

$$\mathbf{M} = \begin{pmatrix} \mathbf{0} & \mathbf{M}_{\nu}^{(\text{Dir})} \\ (\mathbf{M}_{\nu}^{(\text{Dir})})^T & \mathbf{M}_{\nu R}^{(\text{Maj})} \end{pmatrix} \quad (61)$$

Using the fact that the non-vanishing entries of the Majorana mass matrix are much larger than those of the Dirac matrix one can approximately block diagonalize the  $6 \times 6$  matrix and obtain the see-saw matrix [39]

$$\mathbf{M}_{\nu}^{\text{light}} \approx -\mathbf{M}_{\nu}^{(\text{Dir})} (\mathbf{M}_{\nu R}^{(\text{Maj})})^{-1} (\mathbf{M}_{\nu}^{(\text{Dir})})^T \quad (62)$$

as well as

$$\mathbf{M}_{\nu}^{\text{heavy}} \approx \mathbf{M}_{\nu R}^{(\text{Maj})}. \quad (63)$$

$\mathbf{M}_{\nu}^{\text{light}}$  is symmetric and therefore can be diagonalized using one unitary matrix  $\mathbf{N}_{\nu}$

$$\mathbf{N}_{\nu}^T \mathbf{M}_{\nu}^{\text{light}} \mathbf{N}_{\nu} = \mathbf{M}_{\nu}^{\text{light(D)}} \quad (64)$$

Neutrino oscillations are induced via the leptonic analogue to the CKM matrix

$$\mathbf{U} = \mathbf{E}_L^{\dagger} \mathbf{N}_{\nu} \quad (65)$$

## Appendix II: Effective $SO(10)$ Lagrangian for nucleon decays

The baryon number violating part of the  $SO(10)$  Lagrangian (without fermionic mixing) is known to be [4]

$$\begin{aligned} \mathcal{L}_{\Delta B \neq 0} = & \frac{g_U}{\sqrt{2}} \bar{X}_{\mu}^{\alpha} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^{\mu} u_L^{\beta} + \bar{d}_{L\alpha} \gamma^{\mu} e_L^{+} + \bar{d}_{R\alpha} \gamma^{\mu} e_R^{+}) \\ & + \frac{g_U}{\sqrt{2}} \bar{Y}_{\mu}^{\alpha} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^{\mu} d_L^{\beta} - \bar{d}_{R\alpha} \gamma^{\mu} \nu_{eR}^C - \bar{u}_{L\alpha} \gamma^{\mu} e_L^{+}) \\ & + \frac{g_U}{\sqrt{2}} X_{\mu}^{\prime\alpha} (-\varepsilon_{\alpha\beta\gamma} \bar{d}_L^{C\gamma} \gamma^{\mu} d_L^{\beta} - \bar{u}_{L\alpha} \gamma^{\mu} \nu_{eL}^C - \bar{u}_{R\alpha} \gamma^{\mu} \nu_{eR}^C) \\ & + \frac{g_U}{\sqrt{2}} Y_{\mu}^{\prime\alpha} (\varepsilon_{\alpha\beta\gamma} \bar{d}_L^{C\gamma} \gamma^{\mu} u_L^{\beta} - \bar{d}_{L\alpha} \gamma^{\mu} \nu_{eL}^C - \bar{u}_{R\alpha} \gamma^{\mu} e_R^{+}) \\ & + \frac{g_U}{\sqrt{2}} X_{3\mu}^{\alpha} (\bar{d}_{L\alpha} \gamma^{\mu} e_L^{-} + \bar{d}_{R\alpha} \gamma^{\mu} e_R^{-} + \bar{u}_{L\alpha} \gamma^{\mu} \nu_{eL} + \bar{u}_{R\alpha} \gamma^{\mu} \nu_{eR}) \\ & + \text{h.c.} \end{aligned} \quad (66)$$



Taking all possible fermion mixings into account one obtains [40] the effective four fermion Lagrangian

$$\begin{aligned}
\mathcal{L}_{\text{eff}} = & A_1 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{e}_L^+ \gamma_\mu d_L^\alpha) + A_2 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{e}_R^+ \gamma_\mu d_R^\alpha) \\
& + A_3 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{\mu}_L^+ \gamma_\mu d_L^\alpha) + A_4 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{\mu}_R^+ \gamma_\mu d_R^\alpha) \\
& + A_5 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{e}_L^+ \gamma_\mu s_L^\alpha) + A_6 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{e}_R^+ \gamma_\mu s_R^\alpha) \\
& + A_7 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{\mu}_L^+ \gamma_\mu s_L^\alpha) + A_8 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu u_L^\beta) (\bar{\mu}_R^+ \gamma_\mu s_R^\alpha) \\
& + A_9 (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu d_L^\beta) (\bar{\nu}_{eR}^C \gamma_\mu d_R^\alpha) + A_{10} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu d_L^\beta) (\bar{\nu}_{\mu R}^C \gamma_\mu d_R^\alpha) \\
& + A_{11} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu d_L^\beta) (\bar{\nu}_{eR}^C \gamma_\mu s_R^\alpha) + A_{12} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu d_L^\beta) (\bar{\nu}_{\mu R}^C \gamma_\mu s_R^\alpha) \\
& + A_{13} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu s_L^\beta) (\bar{\nu}_{eR}^C \gamma_\mu d_R^\alpha) + A_{14} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu s_L^\beta) (\bar{\nu}_{\mu R}^C \gamma_\mu d_R^\alpha) \\
& + A_{15} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu d_L^\beta) (\bar{\nu}_{\tau R}^C \gamma_\mu d_R^\alpha) + A_{16} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu d_L^\beta) (\bar{\nu}_{\tau R}^C \gamma_\mu s_R^\alpha) \\
& + A_{17} (\varepsilon_{\alpha\beta\gamma} \bar{u}_L^{C\gamma} \gamma^\mu s_L^\beta) (\bar{\nu}_{\tau R}^C \gamma_\mu d_R^\alpha) \\
& + \text{ ( terms with two } s \text{ quarks )} \\
& + \text{ ( terms with } c, b \text{ and } t \text{ quarks )} \\
& + \text{ ( terms with } \bar{\tau}_{L,R}^+ \text{ and } \bar{\nu}_{e,\mu,\tau L}^C \text{ )} \\
& + \text{ h.c.}
\end{aligned} \tag{67}$$

where the coefficients  $A_i$  are given as follows [25]:

$$\begin{aligned}
A_1 &= \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_R)_{11}(\mathbf{D}_L)_{11} + (\mathbf{E}_R)_{21}(\mathbf{D}_L)_{21} + (\mathbf{E}_R)_{31}(\mathbf{D}_L)_{31}) \\
&+ \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{D}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{D}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{D}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{E}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{E}_R)_{31}(\mathbf{U}_L)_{31}) \\
A_2 &= \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_L)_{11}(\mathbf{D}_R)_{11} + (\mathbf{E}_L)_{21}(\mathbf{D}_R)_{21} + (\mathbf{E}_L)_{31}(\mathbf{D}_R)_{31}) \\
&+ \tilde{G}' ((\mathbf{D}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{D}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{D}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_L)_{11}(\mathbf{U}_R)_{11} + (\mathbf{E}_L)_{21}(\mathbf{U}_R)_{21} + (\mathbf{E}_L)_{31}(\mathbf{U}_R)_{31}) \\
A_3 &= \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_R)_{12}(\mathbf{D}_L)_{11} + (\mathbf{E}_R)_{22}(\mathbf{D}_L)_{21} + (\mathbf{E}_R)_{32}(\mathbf{D}_L)_{31}) \\
&+ \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{D}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{D}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{D}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_R)_{12}(\mathbf{U}_L)_{11} + (\mathbf{E}_R)_{22}(\mathbf{U}_L)_{21} + (\mathbf{E}_R)_{32}(\mathbf{U}_L)_{31}) \\
A_4 &= \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_L)_{12}(\mathbf{D}_R)_{11} + (\mathbf{E}_L)_{22}(\mathbf{D}_R)_{21} + (\mathbf{E}_L)_{32}(\mathbf{D}_R)_{31}) \\
&+ \tilde{G}' ((\mathbf{D}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{D}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{D}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_L)_{12}(\mathbf{U}_R)_{11} + (\mathbf{E}_L)_{22}(\mathbf{U}_R)_{21} + (\mathbf{E}_L)_{32}(\mathbf{U}_R)_{31}) \\
A_5 &= \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{U}_L)_{31}) \\
&\quad \cdot ((\mathbf{E}_R)_{11}(\mathbf{D}_L)_{12} + (\mathbf{E}_R)_{21}(\mathbf{D}_L)_{22} + (\mathbf{E}_R)_{31}(\mathbf{D}_L)_{32}) \\
&+ \tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{D}_L)_{12} + (\mathbf{U}_R)_{21}(\mathbf{D}_L)_{22} + (\mathbf{U}_R)_{31}(\mathbf{D}_L)_{32}) \\
&\quad \cdot ((\mathbf{E}_R)_{11}(\mathbf{U}_L)_{11} + (\mathbf{E}_R)_{21}(\mathbf{U}_L)_{21} + (\mathbf{E}_R)_{31}(\mathbf{U}_L)_{31})
\end{aligned}$$

$$\begin{aligned}
A_6 &= \tilde{G} ((U_R)_{11}(U_L)_{11} + (U_R)_{21}(U_L)_{21} + (U_R)_{31}(U_L)_{31}) \\
&\quad \cdot ((E_L)_{11}(D_R)_{12} + (E_L)_{21}(D_R)_{22} + (E_L)_{31}(D_R)_{32}) \\
&+ \tilde{G}' ((D_R)_{12}(U_L)_{11} + (D_R)_{22}(U_L)_{21} + (D_R)_{32}(U_L)_{31}) \\
&\quad \cdot ((E_L)_{11}(U_R)_{11} + (E_L)_{21}(U_R)_{21} + (E_L)_{31}(U_R)_{31}) \\
A_7 &= \tilde{G} ((U_R)_{11}(U_L)_{11} + (U_R)_{21}(U_L)_{21} + (U_R)_{31}(U_L)_{31}) \\
&\quad \cdot ((E_R)_{12}(D_L)_{12} + (E_R)_{22}(D_L)_{22} + (E_R)_{32}(D_L)_{32}) \\
&+ \tilde{G} ((U_R)_{11}(D_L)_{12} + (U_R)_{21}(D_L)_{22} + (U_R)_{31}(D_L)_{32}) \\
&\quad \cdot ((E_R)_{12}(U_L)_{11} + (E_R)_{22}(U_L)_{21} + (E_R)_{32}(U_L)_{31}) \\
A_8 &= \tilde{G} ((U_R)_{11}(U_L)_{11} + (U_R)_{21}(U_L)_{21} + (U_R)_{31}(U_L)_{31}) \\
&\quad \cdot ((E_L)_{12}(D_R)_{12} + (E_L)_{22}(D_R)_{22} + (E_L)_{32}(D_R)_{32}) \\
&+ \tilde{G}' ((D_R)_{12}(U_L)_{11} + (D_R)_{22}(U_L)_{21} + (D_R)_{32}(U_L)_{31}) \\
&\quad \cdot ((E_L)_{12}(U_R)_{11} + (E_L)_{22}(U_R)_{21} + (E_L)_{32}(U_R)_{31}) \\
A_9 &= -\tilde{G} ((U_R)_{11}(D_L)_{11} + (U_R)_{21}(D_L)_{21} + (U_R)_{31}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{11}(D_R)_{11} + (N_L)_{21}(D_R)_{21} + (N_L)_{31}(D_R)_{31}) \\
&- \tilde{G}' ((D_R)_{11}(D_L)_{11} + (D_R)_{21}(D_L)_{21} + (D_R)_{31}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{11}(U_R)_{11} + (N_L)_{21}(U_R)_{21} + (N_L)_{31}(U_R)_{31}) \\
A_{10} &= -\tilde{G} ((U_R)_{11}(D_L)_{11} + (U_R)_{21}(D_L)_{21} + (U_R)_{31}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{12}(D_R)_{11} + (N_L)_{22}(D_R)_{21} + (N_L)_{32}(D_R)_{31}) \\
&- \tilde{G}' ((D_R)_{11}(D_L)_{11} + (D_R)_{21}(D_L)_{21} + (D_R)_{31}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{12}(U_R)_{11} + (N_L)_{22}(U_R)_{21} + (N_L)_{32}(U_R)_{31}) \\
A_{11} &= -\tilde{G} ((U_R)_{11}(D_L)_{11} + (U_R)_{21}(D_L)_{21} + (U_R)_{31}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{11}(D_R)_{12} + (N_L)_{21}(D_R)_{22} + (N_L)_{31}(D_R)_{32}) \\
&- \tilde{G}' ((D_R)_{12}(D_L)_{11} + (D_R)_{22}(D_L)_{21} + (D_R)_{32}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{11}(U_R)_{11} + (N_L)_{21}(U_R)_{21} + (N_L)_{31}(U_R)_{31}) \\
A_{12} &= -\tilde{G} ((U_R)_{11}(D_L)_{11} + (U_R)_{21}(D_L)_{21} + (U_R)_{31}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{12}(D_R)_{12} + (N_L)_{22}(D_R)_{22} + (N_L)_{32}(D_R)_{32}) \\
&- \tilde{G}' ((D_R)_{12}(D_L)_{11} + (D_R)_{22}(D_L)_{21} + (D_R)_{32}(D_L)_{31}) \\
&\quad \cdot ((N_L)_{12}(U_R)_{11} + (N_L)_{22}(U_R)_{21} + (N_L)_{32}(U_R)_{31}) \\
A_{13} &= -\tilde{G} ((U_R)_{11}(D_L)_{12} + (U_R)_{21}(D_L)_{22} + (U_R)_{31}(D_L)_{32}) \\
&\quad \cdot ((N_L)_{11}(D_R)_{11} + (N_L)_{21}(D_R)_{21} + (N_L)_{31}(D_R)_{31}) \\
&- \tilde{G}' ((D_R)_{11}(D_L)_{12} + (D_R)_{21}(D_L)_{22} + (D_R)_{31}(D_L)_{32}) \\
&\quad \cdot ((N_L)_{11}(U_R)_{11} + (N_L)_{21}(U_R)_{21} + (N_L)_{31}(U_R)_{31}) \\
A_{14} &= -\tilde{G} ((U_R)_{11}(D_L)_{12} + (U_R)_{21}(D_L)_{22} + (U_R)_{31}(D_L)_{32}) \\
&\quad \cdot ((N_L)_{12}(D_R)_{11} + (N_L)_{22}(D_R)_{21} + (N_L)_{32}(D_R)_{31}) \\
&- \tilde{G}' ((D_R)_{11}(D_L)_{12} + (D_R)_{21}(D_L)_{22} + (D_R)_{31}(D_L)_{32}) \\
&\quad \cdot ((N_L)_{12}(U_R)_{11} + (N_L)_{22}(U_R)_{21} + (N_L)_{32}(U_R)_{31})
\end{aligned}$$

$$\begin{aligned}
A_{15} &= -\tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{D}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{D}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{D}_L)_{31}) \\
&\quad \cdot ((\mathbf{N}_L)_{13}(\mathbf{D}_R)_{11} + (\mathbf{N}_L)_{23}(\mathbf{D}_R)_{21} + (\mathbf{N}_L)_{33}(\mathbf{D}_R)_{31}) \\
&\quad -\tilde{G}' ((\mathbf{D}_R)_{11}(\mathbf{D}_L)_{11} + (\mathbf{D}_R)_{21}(\mathbf{D}_L)_{21} + (\mathbf{D}_R)_{31}(\mathbf{D}_L)_{31}) \\
&\quad \cdot ((\mathbf{N}_L)_{13}(\mathbf{U}_R)_{11} + (\mathbf{N}_L)_{23}(\mathbf{U}_R)_{21} + (\mathbf{N}_L)_{33}(\mathbf{U}_R)_{31}) \\
A_{16} &= -\tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{D}_L)_{11} + (\mathbf{U}_R)_{21}(\mathbf{D}_L)_{21} + (\mathbf{U}_R)_{31}(\mathbf{D}_L)_{31}) \\
&\quad \cdot ((\mathbf{N}_L)_{13}(\mathbf{D}_R)_{12} + (\mathbf{N}_L)_{23}(\mathbf{D}_R)_{22} + (\mathbf{N}_L)_{33}(\mathbf{D}_R)_{32}) \\
&\quad -\tilde{G}' ((\mathbf{D}_R)_{12}(\mathbf{D}_L)_{11} + (\mathbf{D}_R)_{22}(\mathbf{D}_L)_{21} + (\mathbf{D}_R)_{32}(\mathbf{D}_L)_{31}) \\
&\quad \cdot ((\mathbf{N}_L)_{13}(\mathbf{U}_R)_{11} + (\mathbf{N}_L)_{23}(\mathbf{U}_R)_{21} + (\mathbf{N}_L)_{33}(\mathbf{U}_R)_{31}) \\
A_{17} &= -\tilde{G} ((\mathbf{U}_R)_{11}(\mathbf{D}_L)_{12} + (\mathbf{U}_R)_{21}(\mathbf{D}_L)_{22} + (\mathbf{U}_R)_{31}(\mathbf{D}_L)_{32}) \\
&\quad \cdot ((\mathbf{N}_L)_{13}(\mathbf{D}_R)_{11} + (\mathbf{N}_L)_{23}(\mathbf{D}_R)_{21} + (\mathbf{N}_L)_{33}(\mathbf{D}_R)_{31}) \\
&\quad -\tilde{G}' ((\mathbf{D}_R)_{11}(\mathbf{D}_L)_{12} + (\mathbf{D}_R)_{21}(\mathbf{D}_L)_{22} + (\mathbf{D}_R)_{31}(\mathbf{D}_L)_{32}) \\
&\quad \cdot ((\mathbf{N}_L)_{13}(\mathbf{U}_R)_{11} + (\mathbf{N}_L)_{23}(\mathbf{U}_R)_{21} + (\mathbf{N}_L)_{33}(\mathbf{U}_R)_{31})
\end{aligned}$$

We used here the definitions  $\tilde{G} = g_U^2/2M_{X,Y}^2$  and  $\tilde{G}' = g_U^2/2M_{X',Y'}^2$ , where  $M_{X,Y}^2 = M_{X',Y'}^2 \approx M_U^2$  is assumed.

The coefficients  $A_i$  are connected to the hadronic transition amplitudes of the elementary processes responsible for the nucleon decays. The independent amplitudes are given in tables 12 and 13.

Decay process	Lagrangian term	Amplitude ( $\cdot\sqrt{30}$ )	Coefficient
$p \uparrow \rightarrow e_R^+ u^C \uparrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	-4	$A_2$
$p \uparrow \rightarrow e_R^+ u^C \downarrow u \uparrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	-8	$A_2$
$p \uparrow \rightarrow e_R^+ d^C \uparrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	-8	$A_2$
$p \uparrow \rightarrow e_R^+ d^C \downarrow d \uparrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	+2	$A_2$
$p \downarrow \rightarrow e_R^+ u^C \downarrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	-10	$A_2$
$p \downarrow \rightarrow e_R^+ d^C \downarrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	-2	$A_2$
$p \uparrow \rightarrow e_R^+ s^C \uparrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu s_R)$	-8	$A_6$
$p \uparrow \rightarrow e_R^+ s^C \downarrow d \uparrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu s_R)$	+2	$A_6$
$p \downarrow \rightarrow e_R^+ s^C \downarrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu s_R)$	-2	$A_6$
$p \uparrow \rightarrow \nu_{eR}^C d^C \uparrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+4	$A_9$
$p \uparrow \rightarrow \nu_{eR}^C d^C \downarrow u \uparrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	-10	$A_9$
$p \downarrow \rightarrow \nu_{eR}^C d^C \downarrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	-8	$A_9$
$p \uparrow \rightarrow \nu_{eR}^C s^C \uparrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu s_R)$	+4	$A_{11}$
$p \uparrow \rightarrow \nu_{eR}^C s^C \downarrow u \uparrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu s_R)$	+2	$A_{11}$
$p \uparrow \rightarrow \nu_{eR}^C s^C \uparrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu s_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	0	$A_{13}$
$p \uparrow \rightarrow \nu_{eR}^C s^C \downarrow u \uparrow$	$(\bar{u}_L^C \gamma^\mu s_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	-12	$A_{13}$
$p \downarrow \rightarrow \nu_{eR}^C s^C \downarrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu s_R)$	+4	$A_{11}$
$p \downarrow \rightarrow \nu_{eR}^C s^C \downarrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu s_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	-12	$A_{13}$

Table 12: Decay amplitudes for the elementary processes of proton decays

## References

Decay process	Lagrangian term	Amplitude ( $\cdot\sqrt{30}$ )	Coefficient
$n \uparrow \rightarrow e_R^+ u^C \uparrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	-4	$A_2$
$n \uparrow \rightarrow e_R^+ u^C \downarrow d \uparrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	+10	$A_2$
$n \downarrow \rightarrow e_R^+ u^C \downarrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu u_L)(\bar{e}_R^+ \gamma_\mu d_R)$	+8	$A_2$
$n \uparrow \rightarrow \nu_{eR}^C u^C \uparrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+8	$A_9$
$n \uparrow \rightarrow \nu_{eR}^C u^C \downarrow u \uparrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	-2	$A_9$
$n \uparrow \rightarrow \nu_{eR}^C d^C \uparrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+4	$A_9$
$n \uparrow \rightarrow \nu_{eR}^C d^C \downarrow d \uparrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+8	$A_9$
$n \downarrow \rightarrow \nu_{eR}^C u^C \downarrow u \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+2	$A_9$
$n \downarrow \rightarrow \nu_{eR}^C d^C \downarrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+10	$A_9$
$n \uparrow \rightarrow \nu_{eR}^C s^C \uparrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu s_R)$	+4	$A_{11}$
$n \uparrow \rightarrow \nu_{eR}^C s^C \downarrow d \uparrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu s_R)$	-4	$A_{11}$
$n \uparrow \rightarrow \nu_{eR}^C s^C \uparrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu s_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	0	$A_{13}$
$n \uparrow \rightarrow \nu_{eR}^C s^C \downarrow d \uparrow$	$(\bar{u}_L^C \gamma^\mu s_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+12	$A_{13}$
$n \downarrow \rightarrow \nu_{eR}^C s^C \downarrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu d_L)(\bar{\nu}_{eR}^C \gamma_\mu s_R)$	-2	$A_{11}$
$n \downarrow \rightarrow \nu_{eR}^C s^C \downarrow d \downarrow$	$(\bar{u}_L^C \gamma^\mu s_L)(\bar{\nu}_{eR}^C \gamma_\mu d_R)$	+12	$A_{13}$

Table 13: Decay amplitudes for the elementary processes of neutron decays

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